A Fuzzy Relational Approach to Event Recommendation

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Abstract. Most existing recommender systems employ collaborative filtering (CF) techniques in making projections about which items an e-service user is likely to be interested in, i.e. they identify correlations between users and recommend items which similar users have liked in the past. Traditional CF techniques, however, have difficulties when confronted with sparse rating data, and cannot cope at all with time-specific items, like events, which typically receive their ratings only after they have finished. Content-based (CB) algorithms, which consider the internal structure of items and recommend items similar to those a user liked in the past can partly make up for that drawback, but the collaborative feature is totally lost on them. In this paper, modelling user and item similarities as fuzzy relations, which allow to flexibly reflect the graded/uncertain information in the domain, we develop a novel, hybrid CF-CB approach whose rationale is concisely summed up as "recommending future items if they are similar to past ones that similar users have liked", and which surpasses related work in the same spirit.

1 Introduction

Personalization applications, and more in particular recommender systems, have gained momentum in the e-service domain over the past years as a means of understanding, and catering to, the specific needs of individuals or groups of customers, and of resolving the problem of information overload. Recommender systems attempt to predict which items (e.g. retail products, websites, services,etc.) are likely to be of interest to a user. To help these applications to make such projections, an eclectic variety of information sources is generally available, ranging from past purchase records, explicit ratings of items, social network information, to object hierarchies and user profiles. More often than not, these data are inherently graded (such as ratings expressing satisfaction/dissatisfaction with a

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product), or fraught with uncertainties (such as the identification of users similar to an individual about which only low-quality information is available). For this reason, fuzzy relations, and the elaborate machinery designed to combine and propagate them, can offer an attractive solution to modelling, implementing and evaluating the process of recommendation.

In this paper, specifically, we consider three fuzzy relations: one between users and one between items (expressing the respective *similarities* among them), and one from users to items (expressing people's *preferences*). In that sense, the paper extends the seminal work in this direction by Perny and Zucker [12, 13]. The proposed approach, which offers system developers a richer palette of relational compositions to choose from, is in particular relevant to deal with the problem of sparse rating data, i.e. when an item has not received enough evaluations to be meaningfully used in collaborative filtering (CF) recommendations. In the limit case, we have to decide whether to recommend items that have not been rated at all; this applies in particular to event recommendation. As ephemeral items, events, once they have taken place, are meaningless to recommend, but it is only at that time that people who have attended them usually submit their evaluations. A conventional content-based (CB) system could step in here to recommend future events that are similar (using an adequate comparison metric) to ones a user has liked in the past, but such an approach implies that 1) the user has in fact attended sufficient past events and 2) only events in the similarity neighbourhood of *this* user's past preference record could ever be recommended. A collaborative feature, positioning a user within a network of related individuals, and allowing them to explore new areas those fellow users have appreciated, is lacking. Our approach aims to fill that gap by effectively encapsulating and integrating the CB and CF paradigms in a novel, flexible and robust way, namely by recommending an item to a user inasfar as it is similar (or equal) to items that this user, or similar users[1], have liked in the past. As we will demonstrate, this allows the recommendation strategy to optimally identify, and exploit, relations existing in the domain. Our scheme also provides a way of presenting positive and negative feelings separately, and allows to deal with information shortage and information excess.

The remainder of this paper is organized as follows: in Section 2, we recall the necessary preliminaries from fuzzy set theory and fuzzy relational calculus. Section 3 reviews existing work on recommender systems and their shortcomings. The emphasis is on Perny and Zucker's framework from [13] because it acts as the starting point for our approach, which is presented in detail in Section 4. In Section 5, we offer a brief conclusion and point out future work, including an envisaged application of the algorithm used by our approach in the practical setting of e-government.

[1] Here, we consider similarity between users in a broad sense, including also e.g. assessments of the competence of one user as an advisor to another one.

2 Preliminaries

Fuzzy sets were introduced by Zadeh in 1965 in [18]. A fuzzy set A in a universe U is seen as a mapping from U to the unit interval $[0, 1]$. For any u in U, the value $A(u)$ is called the membership degree of u to A. It is clear that any crisp, or classical, set A in U can be seen as a fuzzy set in which the membership degrees are restricted to 0 (non-membership) and 1 (membership). In this paper, we will only consider fuzzy sets in finite universes.

A fuzzy relation R from U to V is a fuzzy set in $U \times V$, where $R(u, v)$ expresses to what extent the elements u in U and v in V are related by R. As a concrete example, we can define a fuzzy preference relation P from U, the universe of users, to I, the universe of items; for each user u and each item i, $P(u, i)$ denotes the degree to which i is preferred, or liked, by u.

Given a fuzzy relation R from U to V, R^{-1}, the inverse fuzzy relation of R, is a fuzzy relation from V to U defined by $R^{-1}(v, u) = R(u, v)$. A fuzzy relation R from U to U is also called a fuzzy relation in U. It is called reflexive if for all u in U, $R(u, u) = 1$ and symmetrical if $R(u, v) = R(v, u)$ for all u and v in U.

Like their crisp counterparts, fuzzy relations can be composed in different ways (see e.g. [8] and [9] for extensive coverage of this topic). We first recall the definitions of triangular norms (shortly, t-norms) and implicators on [0,1], which will be needed for this purpose and which generalize classical conjunction, and implication, respectively.

Definition 1. (t-norm) A triangular norm, or t-norm, $\mathcal{T}$ is a commutative, associative, increasing $[0, 1]^2 \to [0, 1]$ mapping that satisfies $\mathcal{T}(x, 1) = x$ for all x in [0,1].

Definition 2. (Implicator) An implicator $\mathcal{I}$ is a $[0, 1]^2 \to [0, 1]$ mapping with decreasing first and increasing second partial mappings that satisfies $\mathcal{I}(0, 0) = \mathcal{I}(0, 1) = \mathcal{I}(1, 1) = 1$ and $\mathcal{I}(1, 0) = 0$.

Common t-norms include the minimum, the product and the Łukasiewicz t-norm $\mathcal{T}_W$ defined by $\mathcal{T}_W(x, y) = \max(0, x+y-1)$. The implicator $\mathcal{I}_\mathcal{T}$, called the residual implicator of $\mathcal{T}$, is defined by $\mathcal{I}_\mathcal{T}(x, y) = \sup\{\gamma \in [0, 1] \mid \mathcal{T}(x, \gamma) \leq y\}$. For instance, the residual implicator $\mathcal{I}_{\mathcal{T}_W}$ of $\mathcal{T}_W$ is defined by $\mathcal{I}_{\mathcal{T}_W}(x, y) = \min(1, 1-x+y)$ and is called the Łukasiewicz implicator. It is a common choice in many applications as it inherits many useful properties of its classical counterpart.

Definition 3. (Composition of fuzzy relations) Let R be a fuzzy relation from U to V, and S a fuzzy relation from V to W, $\mathcal{T}$ a t-norm and $\mathcal{I}$ an implicator. Then the sup-$\mathcal{T}$-composition $R \circ_\mathcal{T} S$, the subproduct $R \triangleleft_\mathcal{I} S$ and the superproduct $R \triangleright_\mathcal{I} S$ are defined by, for all u, w in $U \times W$,

$$R \circ_\mathcal{T} S(u, w) = \sup_{v \in V} \mathcal{T}(R(u, v), S(v, w)) \tag{1}$$

$$R \triangleleft_\mathcal{I} S(u, w) = \inf_{v \in V} \mathcal{I}(R(u, v), S(v, w)) \tag{2}$$

$$R \triangleright_\mathcal{I} S(u, w) = \inf_{v \in V} \mathcal{I}(S(v, w), R(u, v)) \tag{3}$$

The meaning of these compositions is as follows: (u, w) belongs to $R \circ_{\mathcal{T}} S$ to the extent that there exists a v in V such that u is related by R to v and v is related by S to w, to $R \triangleleft_{\mathcal{I}} S$ to the degree that *every* v to which u is related by R, is related by S to w, and to $R \triangleright_{\mathcal{I}} S$ insofar as for every v which is related by S to w it holds that u is also related by R to v.

For example, let R denote a symmetrical fuzzy relation in U that expresses similarity between the users of a given e-service, and let P be the above-defined fuzzy preference relation. Then the sup-$\mathcal{T}$-composition $R \circ_{\mathcal{T}} P$ expresses for each couple (u, i) in $U \times I$, to what degree there exists a similar user to u who likes i; taking the subcomposition $R \triangleleft_{\mathcal{I}} P$ allows us to identify the items that are liked by all users that are similar to u, while (u, i) belongs to $R \triangleright_{\mathcal{I}} P$ to the extent that i is *only* preferred by users similar to u.

Remark 1. Note that while the above reflects the general meaning conveyed by the respective compositions, the choice of the implicator $\mathcal{I}$ and the t-norm $\mathcal{T}$ plays an important role and has to be considered carefully in view of the application at hand. As an example, using $\mathcal{T} = \mathcal{T}_W$ in (1), will cause weak links between u and w through elements of V not to be taken into account. For instance, if $R(u, v) = S(v, w) = 0.5$, then still $\mathcal{T}_W(R(u, v), S(v, w)) = 0$ whereas for the alternative choice $\mathcal{T} = \min$, we find $\min(R(u, v), S(v, w)) = 0.5$. For an in-depth study on t-norms and their practical usage, please refer to e.g. [11].

On another count, for practical applications, the relational compositions from Definition 3 may sometimes considered too harsh because of the use of sup and inf. For instance, $R \triangleleft_{\mathcal{I}} S(u, w) = 0$ as soon as there exists a single v such that $R(u, v) = 1$ and $S(v, w) = 0$. In such cases, it is worthwhile to mellow down the compositions (1)–(3) by taking a weighthed average of values, instead of the best or worst ones, to obtain the alternative formulas

$$R \circ^a_{\mathcal{T}} S(u, w) = \frac{\sum_{v \in V} c_v \mathcal{T}(R(u, v), S(v, w))}{\sum_{v \in V} c_v} \quad (4)$$

$$R \triangleleft^a_{\mathcal{I}} S(u, w) = \frac{\sum_{v \in V} c_v \mathcal{I}(R(u, v), S(v, w))}{\sum_{v \in V} c_v} \quad (5)$$

$$R \triangleright^a_{\mathcal{I}} S(u, w) = \frac{\sum_{v \in V} c_v \mathcal{I}(S(v, w), R(u, v))}{\sum_{v \in V} c_v} \quad (6)$$

In these formulas, c_v represents the weight (a value in $[0, 1]$) of each v in V.

3 Related Work

The main strategies of existing recommender systems are content-based (CB) and collaborative filtering (CF). The content-based approach recommends objects that are similar to those in which the user has been interested in the past.

It originally derived from machine learning research and has been adopted by the information retrieval society, where textual documents are recommended by comparing their contents and user profiles. [16] Often some weighting schemes are used which give high weights to discriminating words. Once an object has been picked, it can be shown to users and feedback of some kind elicited; if the user likes the object, the weight of the object can be added to the user profile. More generally, the CB approach can be taken to encompass any recommendation scheme that involves an internal representation of the items (often by a vector of numerical values, as e.g. in the semantic product relevance model developed in [6]).

CF is the most popular recommendation technique used in various different applications, such as the recommendation of web pages, movies, articles, etc. (see e.g. [7, 15]). The CF-based approach identifies users whose tastes are similar to those of the given user and recommends items they have liked. A recommender system can compare a target user's ratings to those of other users to find the "most similar" users based on some criteria of similarity, and then recommend items that similar users have liked in the past. Scores for unseen items are predicted based on a combination of the scores known from the nearest neighbours.

Given the proliferance of weighting schemes (to qualify e.g. a user's appreciation of an item, or the importance of a user as an advisor for another user) in the existing approaches, topped by the fact that very often decisions have to be made under incomplete and/or conflicting information, it comes as quite a surprise that fuzzy set theory, with its well-earned reputation for negotiating and even exploiting imperfections to produce lower-cost, better-quality solutions, has not taken up a more prominent role in the literature on recommender systems. Indeed, the intricate patchwork of correlation and preference representation and aggregation processes encountered in recommender systems positions itself neatly between the well-researched domains of individual multiple-attribute fuzzy decision making, and group negotiation. This was noticed by Perny and Zucker [12, 13], who coined the term "Collaborative Decision Support" (CDS) to denote decision making problems where individuals seek recommendations for their personal choices, the other individuals being considered as possible advisors. They pursued a hybrid approach that involves a content-based and a collaborative filtering component. In particular, they considered the following fuzzy relations:

- P^+, from U to I, expresses *positive feelings* (satisfaction) of a user about an item
- P^-, from U to I, expresses *negative feelings* (disgust) of a user about an item
- S, in I, expresses *similarity* between items
- R, in U, expresses *similarity* between users
- Q, from U to I, expresses the *qualification* of a user w.r.t. his rating of an item
- $\hat{P}$, from U to I, expresses the *predicted preference* of a user for an item

P^+ and P^- are derived from the actual preference information (e.g., ratings) by a suitable transformation, where $\min(P^+(u,i), P^-(u,i)) = 0$ is imposed for each couple (u,i) in $U \times I$, indicating that user u is either positively, or negatively, inclined about item i. Using appropriate fuzzy similarity measures, for each item i, and each user u, a neighbourhood of k most similar elements is constructed and denoted $N_k(i)$, respectively $N_k(u)$; thanks to the use of neighbourhoods, the entire search space need not be traversed in producing recommendations. Next, $Q(u,i)$ can be a self- or peer-evaluation of the confidence about u's rating of i, to strengthen or diminish its impact in the generation of recommendations. Finally, the target relation $\hat{P}$ is computed by the following formula, for u in U and i in I:

$$\hat{P}(u,i) = (1-\beta)\hat{P}_{CB}(u,i) + \beta\hat{P}_{CF}(u,i) \tag{7}$$

where β is a weighting parameter in $[0,1]$ and $\hat{P}_{CB}(u,i)$ and $\hat{P}_{CF}(u,i)$ are the content-based, respectively collaborative filtering components to the recommendation, defined by, for a t-norm $\mathcal{T}$,

$$\hat{P}_{CB}(u,i) = \mathcal{T}\left(P^+_{CB}(u,i), 1 - P^-_{CB}(u,i)\right) \tag{8}$$

$$\hat{P}_{CF}(u,i) = \mathcal{T}\left(P^+_{CF}(u,i), 1 - P^-_{CF}(u,i)\right) \tag{9}$$

$$\hat{P}^+_{CB}(u,i) = \sup_{j \in N_k(i)} \mathcal{T}(P^+(u,j), S(j,i)) \tag{10}$$

$$\hat{P}^-_{CB}(u,i) = \sup_{j \in N_k(i)} \mathcal{T}(P^-(u,j), S(j,i)) \tag{11}$$

$$\hat{P}^+_{CF}(u,i) = \sup_{v \in N_k(u)} \mathcal{T}(\mathcal{T}(Q(v,i), P^+(v,i)), R(v,u)) \tag{12}$$

$$\hat{P}^-_{CF}(u,i) = \sup_{v \in N_k(u)} \mathcal{T}(\mathcal{T}(Q(v,i), P^-(v,i)), R(v,u)) \tag{13}$$

With these definitions, user u receives a high recommendation on item i if

- i is similar to any j_1 which u likes and is not similar to any j_2 which u dislikes {formulas (8) and (10)–(11)}.
- i is liked by a given v_1 who is similar to u, and there doesn't exist any v_2 similar to u who dislikes i; the appearance of $Q(v,i)$ in the formulas is to ensure the authority of v's evaluation of i {formulas (9) and (12)–(13)}.

Noting that the occurence of a single outspokenly positive or negative evaluation in (10)–(13) can affect the final outcome dramatically, they proposed mellowed versions of these formulas akin to the weighted relational compositions (4)–(6). They furthermore fine-tuned and refined their recommendation algorithm by thresholding and machine learning techniques, and implemented it in the "Film-Conseil" movie recommender system.

To conclude this section, we should also mention the work of Yager [17], who has devised a sophisticated fuzzy recommendation scheme that allows to model users' preferences (both expressed intentionally, i.e. through explicit specifications on users' behalf, and extensionally, that is, based on their past actions and experiences). The proposed recommendation strategies are however purely content-based (or reclusive, as the author calls them).

4 Recommendation Algorithm

Perny and Zucker's framework, while quite elaborate and flexible in itself, has some important drawbacks:

- It is hard to set an appropriate value for the parameter β balancing the impact of content-based and collaborative filtering contributions to the final recommendation $\hat{P}(u,i)$.
- The CF component is useless for recommending items that have not been rated yet; that is, for such items, their approach is totally reliant on pure CB recommendation.

The former is a limitation affecting hybrid CB-CF recommender systems in general. The latter is especially problematic to event recommendation. For this reason, an extension of their framework is proposed. To improve the clarity and simplicity of the exposition, we will maintain the same symbols to denote the various fuzzy relations that play a role in the recommendation process, and will not use the qualification relation Q, which can in fact be easily absorbed into the definitions of P^+ and P^- by putting

$$P^+(u,i) := \mathcal{T}(P^+(u,i), Q(u,i)) \tag{14}$$

$$P^-(u,i) := \mathcal{T}(P^-(u,i), Q(u,i)) \tag{15}$$

with $\mathcal{T}$ a t-norm. In the following, we outline the most important details of our recommendation strategy.

Preference modelling. We lift the restriction that $\min(P^+(u,i), P^-(u,i)) = 0$, because it splits up users into crisp "pro" and "contra" sides w.r.t. any given item, while generally there is a smooth transition between the two camps. In general, we allow any couple $(P^+(u,i), P^-(u,i))$ of values in $[0,1]^2$ to express a user's feelings w.r.t. an item. The following "special" values are distinguished:

$(1,0) \rightarrow$ Outspoken preference. u likes i very much and would readily recommend it to others.
$(0,1) \rightarrow$ Outspoken disgust. u hates i.
$(0,0) \rightarrow$ Ignorance. u has not experienced, or expressed any opinion about, i.
$(1,1) \rightarrow$ Conflict. u has expressed contradicting opinions about i.
$\left(\frac{1}{2}, \frac{1}{2}\right) \rightarrow$ Neutrality. The negative and positive arguments weigh up to each other.

It is important to stress that these two-sided evaluations are cognitive states reflecting the algorithm's *knowledge* about the user's preferences; each component captures the evidence gathered by the algorithm through explicit or implicit user querying about the item. As such, they fit in with the bilattice-based, $[0,1]^2$-valued approach to representing imprecise information, discussed in e.g. [1]. To quantify the amount of information we have about the evaluation of i by u, we introduce the $[0,1]$-valued measure $\mathcal{K}$ ("knowledge"), given by $\mathcal{K}(u,i) =$

$\min(1, P^+(u,i) + P^-(u,i))$. Then $\mathcal{K}(u,i) = 0$ if $(P^+(u,i), P^-(u,i)) = (0,0)$, and $0 < \mathcal{K}(u,i) \leq 1$ in all other situations.

Similarity. For comparison purposes, we assume that each item has associated with it a number of descriptive attributes. It is clear that as this description gets finer and more accurate, a measure of similarity (also: resemblance, proximity) will have more discriminating power. Computing $[0,1]$-valued similarity between items described by attribute vectors is a problem well-covered in literature, see e.g. [3]. We assume here that the fuzzy relation S in I is at least reflexive (an items resembles itself perfectly) and symmetrical (the order in which items are compared is irrelevant).

User similarity is more complicated. While it is technically possible to pursue the same approach as for items by comparing user description (profile) vectors, this is often impractical because in many e-services users are quasi anonymous and can only be assessed based on their actions. Moreover, the relation R we are looking for need not necessarily be symmetrical, as in collaborative filtering the role of fellow users is primarily as advisors who can direct the target user to interesting items. Noting this, Perny and Zucker [13] considered, amongst others, "positive influence" of u over v, by evaluating to what degree everything u (dis)likes, v (dis)likes too. This amounts to the following formula:

$$R(u,v) = \mathcal{T}\left(\inf_{i \in I} \mathcal{I}(P^+(u,i), P^+(v,i)), \inf_{i \in I} \mathcal{I}(P^-(u,i), P^-(v,i))\right) \quad (16)$$

$$= \mathcal{T}\left((P^+ \triangleleft_{\mathcal{I}} (P^+)^{-1})(u,v), (P^- \triangleleft_{\mathcal{I}} (P^-)^{-1})(u,v)\right) \quad (17)$$

where $\mathcal{T}$ is a t-norm and $\mathcal{I}$ is an implicator. The rationale of this formula can be understood if one recalls that t-norms and implicators extend classical conjunction and implication, respectively. A drawback of this formula, and of existing CF approaches in general, is that by focussing strictly on those items that both u and v have rated, they tend to overlook a lot of interesting relationships in the domain. For instance, suppose that I is a book collection, and that u has positively evaluated novels by Edgar Allan Poe and Mary Shelley, and that v likes "Wuthering heights" by Emily Bronte (but didn't rate any of the books u read) hence both of them seem to be fond of gothic literature, and u might make a great advisor for v. However, since they have no rated items in common, the CF algorithm cannot discover this shared interest! To circumvent this problem, we can reformulate the user similarity criterion as "for every item i that u likes, there exists a *similar* item that v likes". In terms of fuzzy relations, this means we have to evaluate the sup-$\mathcal{T}$-compositions $S \circ_{\mathcal{T}} (P^+)^{-1}$ and $S \circ_{\mathcal{T}} (P^-)^{-1}$ instead of $(P^+)^{-1}$ and $(P^-)^{-1}$, and the formulas (16) and (17) can be replaced by

$$
\begin{aligned}
R(u,v) = \mathcal{T}\Bigg(&\inf_{i\in I}\mathcal{I}(P^+(u,i), \sup_{j\in I}\mathcal{T}(S(i,j), P^+(v,j)))\ ,\\
&\inf_{i\in I}\mathcal{I}(P^-(u,i), \sup_{j\in I}\mathcal{T}(S(i,j), P^-(v,j)))\Bigg) && (18)\\
= \mathcal{T}\big(&(P^+ \triangleleft_{\mathcal{I}} (S \circ_{\mathcal{T}} (P^+)^{-1}))(u,v),\\
&(P^- \triangleleft_{\mathcal{I}} (S \circ_{\mathcal{T}} (P^-)^{-1}))(u,v)\big) && (19)
\end{aligned}
$$

Proposition 1. For every t-norm $\mathcal{T}$, and every elements u and v in U, the value of (19) is at least as high as that of (17).

Proposition 1 means that any link discovered through classical CF, will also be found with our approach, so (19) is truly an extension of (17). We can further custom-tailor this formula by replacing the harsh subcomposition in it by a weighted average composition, that is:

$$
\begin{aligned}
R(u,v) = \mathcal{T}\Bigg(&\frac{\sum_{i\in I} c_i\mathcal{I}(P^+(u,i), \sup_{j\in I}\mathcal{T}(S(i,j), P^+(v,j)))}{\sum_{i\in I} c_i}\ ,\\
&\frac{\sum_{i\in I} c_i\mathcal{I}(P^-(u,i), \sup_{j\in I}\mathcal{T}(S(i,j), P^-(v,j)))}{\sum_{i\in I} c_i}\Bigg) && (20)\\
= \mathcal{T}\big(&(P^+ \triangleleft^a_{\mathcal{I}} (S \circ_{\mathcal{T}} (P^+)^{-1}))(u,v),\\
&(P^- \triangleleft^a_{\mathcal{I}} (S \circ_{\mathcal{T}} (P^-)^{-1}))(u,v)\big) && (21)
\end{aligned}
$$

The weights c_i can be chosen to reflect the importance of each item in the overall user similarity evaluation. For instance, if i, nor anything remotely similar to i, was rated by v, this item should not have an impact on $R(u,v)$. We can obtain this behaviour by putting[2]

$$c_i = \sup_{j\in I}\mathcal{T}(S(i,j), \mathcal{K}(v,j)) \tag{22}$$

Finally, for efficiency purposes, it is better to replace I in (20) by $N_k(i)$ so that only a close neighbourhood of each item i is considered during user similarity evaluation (distant items j cannot make a substantial contribution to the outcome anyway because of the low value of $S(i,j)$).

Recommendation formula. Proceeding in a similar vein as above, making optimal use of the information that the fuzzy relations R and S provide, we replace Perny and Zucker's recommendation formulas (8)-(13) by the following ones:

[2] Technically, this choice of the weights is not in line with formula (5) because c_i depends also on v, but for practical purposes this does not pose any problem.

$$\hat{P}(u,i) = \left(\hat{P}^+(u,i), 1 - \hat{P}^-(u,i))\right) \tag{23}$$

$$\hat{P}^+(u,i) = \sup_{v \in N_k(u)} \mathcal{T}\left(\sup_{j \in N_k(i)} \mathcal{T}(S(i,j), P^+(v,j)), R(v,u)\right) \tag{24}$$

$$\hat{P}^-(u,i) = \sup_{v \in N_k(u)} \mathcal{T}\left(\sup_{j \in N_k(i)} \mathcal{T}(S(i,j), P^-(v,j)), R(v,u)\right) \tag{25}$$

Proposition 2. For every t-norm $\mathcal{T}$ and implicator $\mathcal{I}$, and every elements u in U and i in I, the value of (24) is at least as high as that of (10) and (12), and the value of (25) is at least as high as that of (11) and (13).

Proposition 2 implies that formulas (24) and (25) encompass at the same time the content-based ánd collaborative filtering paradigms. The need for choosing an appropriate β value to balance between the two components vanishes, because whichever of them has the stronger impact prevails! Moreover, these formulas allow for useful recommendations that the existing hybrid approach could never come up with: if u is similar to v, and v likes j which is similar to i, then i can be considered interesting for u. Not only does this idea allow the algorithm to explore new regions in the search space, it is in particular relevant to event recommendation, for which the classical CF formula (9) does not yield any useful result. As evidenced by (24)–(25), in our setting users *can* influence each other even in this special situation.

Just like before, these formulas can be straightforwardly replaced by weighted averages. Note that in all cases the final outcome (23) of the algorithm is two-valued: $\hat{P}(u,i)$ juxtaposes the arguments in favour of, respectively against, recommending u to i. If necessary, a suitable transformation to the linearly ordered scale $[0,1]$ can be picked; this is the case, for example, if a top-N recommendation is to be obtained from it, that is: a set of N most attractive items among the (unseen) elements of I for a particular user has to be constructed[3].

5 Conclusions and Future Work

This paper has introduced a new conceptual approach for the recommendation, in the context of e-services, of items about which we only assumed that a suffi-

[3] Simply using a t-norm as in formula (7), or an average value, might have some adverse effects, because it disregards the amount of information (shortage or excess) that the algorithm has been able to gather about this particular user and item. In [4], some useful and more elaborate scoring procedures, tailored to $[0,1]^2$ positive/negative preference evaluations, are proposed. Otherwise, some common-sense "fuzzy" reasoning may be applied, such as to increase the eligibility of an item for recommendation as its $\hat{P}^+$ component gets much greater than its $\hat{P}^-$ component. This remains an object of ongoing research.

ciently rich internal representation is available. The most important qualities of our approach are:

(1) The use of fuzzy relations and their various compositions allows to optimally capture and exploit the relationships between users and items existing in the domain, which is an enormous asset in the presence of sparse or non-existent rating data (as is often the case with events).
(2) The integration of the collaborative filtering and content-based paradigms into a single formalism presents an elegant, intuitive and unified synthesis of the problem of recommendation.
(3) The two-sided positive/negative evaluations of the predicted preference of a user for an item allow to take the strength of the arguments into account in the recommendation process.

In these respects, our approach surpasses the earlier work of Perny and Zucker on fuzzy recommender systems from which we have started.

For the future, we plan to validate our algorithm on large-scale datasets in an effort to meet the demands of realistic applications. In particular, in a forthcoming paper, we consider a particular government-to-business personalization application concerned with the recommendation of international trade exhibition events (see also [6]). These events are frequently used in exporting firms' marketing strategies and are of great value for exporting firms to communicate with potential and current customers from many countries in a short period of time. We believe that the recommendation techniques described in this paper can be instrumental in offering governments the means to satisfy interests and needs of particular business users.

A further refinement and fine-tuning of our approach will also involve the study of generalized compositions of fuzzy relations, based on the ordered weighted averaging operators used by Yager in [17], as well as of suitable procedures for ranking the two-valued prediction values according to their fitness for recommendation (as in top-N recommendation). In terms of complexity, our algorithm's increased search ability and expressiveness comes with an extra cost, as the construction of R and $\hat{P}$ require the computation of an additional supremum for any couple of elements, yet this cost can be kept within bounds by dynamically adapting the neighbourhood size according to the need for extra information. This will also be an object of our further investigations.

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