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On (un)suitable fuzzy relations to model approximate equality

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Abstract

In this paper we state that fuzzy equivalence relations in general are not suitable to model approximate equality, since then the notion of transitivity is counter-intuitive. To substantiate this we investigate some of the undesirable results caused by transitivity, among other things in the case of approximate reasoning. We then introduce a new framework to model approximate equality, i.e. the concept of a pseudometric based resemblance relation. We go into the properties of this new kind of fuzzy relation and illustrate it by means of some examples. © 2002 Elsevier Science B.V. All rights reserved.

Keywords: Fuzzy relations; Fuzzy equivalence relations; Resemblance relations; Approximate reasoning

1. Introduction

An *equivalence relation*, i.e. a reflexive, symmetric and transitive relation, can be considered as a basic concept of mathematics. The most popular example of an equivalence relation is the *crisp equality* (see Fig. 1, \leftarrow^1). The fuzzy counterpart of crisp equality is *approximate equality* (Fig. 1, \downarrow_3), while the concept of a crisp equivalence relation can be fuzzified to that of a *fuzzy equivalence relation* (Fig. 1, \downarrow_2). Unlike in the crisp case, however, where crisp equality is intuitively a crisp equivalence relation, we will show that in the fuzzy case approximate equality is intuitively not always a fuzzy equivalence relation. Taking into account the strong intuitive connection between *approximate equality* and *distance*, in this paper we replace the “problematic” condition of transitivity by a pseudometric based condition. Thus we come to the notion of *resemblance relation*, which is a suitable framework to model approximate equality.

After recalling some basic concepts from fuzzy set theory (Section 2), we go into paradoxical results that appear when representing approximate equality by fuzzy equivalence relations, among other things in the case of approximate reasoning (Section 3). Next, a study of the links between

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